

p412

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

$$\textcircled{6} \ln(1-x)$$

$$\ln(1+(-x))$$

$$= -x - \frac{(-x)^2}{2} + \frac{(-x)^3}{3} - \frac{(-x)^4}{4} + \dots$$

$$= -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots - \frac{x^n}{n}$$

$$\text{interval: } -1 < -x \leq 1$$

$$\boxed{1 > x \geq -1}$$

$$\textcircled{8} 7x e^x$$

$$7x \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots \right)$$

$$7x + 7x^2 + \frac{7x^3}{2!} + \dots + \frac{7x^{n+1}}{n!}$$

$$\textcircled{9} \cos(x+2) = (\cos 2)(\underline{\cos x}) - (\sin 2)(\underline{\sin x})$$

$$= (\cos 2) \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots \right) - (\sin 2) \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \right)$$

$$= \cos 2 - (\sin 2)x - (\cos 2)\frac{x^2}{2!} + (\sin 2)\frac{x^3}{3!}$$

(24)

$$f(x) = 1 + \boxed{\frac{x}{2!}} + \frac{x^2}{3!} + \frac{x^3}{4!} + \dots + \frac{x^n}{(n+1)!} + \dots \frac{x^{10}}{11!}$$

$$a) f'(0) = ? \frac{1}{2} \quad \frac{f^{(n)}(0)}{n!} x^n \quad \frac{1}{2} \boxed{f'(0)} x^1$$

$$f^{(10)}(0) = ?$$

$$\frac{1}{11} \boxed{f^{(10)}(0)} x^{10}$$

$$b) g(x) = x \cdot \underline{f(x)}$$

$$= x \left(1 + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} + \dots \right)$$

$$= x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

$$c) e^x - 1$$

9.3 Taylor's Theorem

Find an adequate Taylor Polynomial $P(x)$ for $\sin x$ on $[-\pi, \pi]$.

"adequate" - values using $P(x)$ must be less than 0.0001 off from $\sin x$ on $[-\pi, \pi]$.

Choose n so that $|P_n(x) - \sin x| < 0.0001$
Worst approx at $-\pi, \pi$

$$|P_n(\pi) - \sin \pi| < 0.0001$$

$$|P_n(\pi)| < 0.0001$$

$$P_n(\pi) = \pi - \frac{\pi^3}{3!} + \frac{\pi^5}{5!} - \frac{\pi^7}{7!} + \dots$$

$$P_1(\pi) = \pi$$

$$P_3(\pi) = \pi - \frac{\pi^3}{3!} \approx -2.026$$

$$P_5(\pi) = \pi - \frac{\pi^3}{3!} + \frac{\pi^5}{5!} \approx 0.524$$

$\vdots$

$$P_{13}(\pi) = \pi - \frac{\pi^3}{3!} + \frac{\pi^5}{5!} - \frac{\pi^7}{7!} + \frac{\pi^9}{9!} - \frac{\pi^{11}}{11!} + \frac{\pi^{13}}{13!}$$

HW: p500 # 1, 3, 6, 7, 10
p492 # 37, 38, 39